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Students' Recognition of Concepts of Reflection and Refraction in Multiple Representational Formats

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Abstract – *A complete understanding of reflection and refraction is achieved when students can recognize the concepts in different representations such as verbal, mathematical, and ray diagrams. The study explores the inconsistencies and difficulties in the students' understanding of image formation by mirrors and lenses. The primary researcher analyzed data on students' performance on a 20-item test consisting of verbal, mathematical, and ray diagram representation items. Two hundred thirty-one (231) grade 10 students took the test after 52-hour instructions on light reflection and refraction. The test results reveal a recognition of the concepts of image formation better in verbal representation by the students. In addition, chi-square results implied that students had drawn the rays in their ray diagrams of spherical mirrors based on their equivalent understanding of the situation in verbal representation. Inconsistencies in their knowledge of reflection by mirrors and refraction by lenses were identified by the differences in the students' responses to verbal and ray diagram representations and supported by the number of students who correctly answered the same items in these two representations. Inconsistencies with the mirror or lens equation were also determined by comparing the results of the items in mathematical representation. Students are found to have difficulties applying the mirror or lens equation and ray-tracing method in problem situations. These observed inconsistencies and problems in multiple representations imply that students have a poor and incomplete understanding of the topic under study.*

Keywords: *difficulties; inconsistencies; multiple representations; understanding*

I. INTRODUCTION

Instructions in physics have varied in multiple representations, such as verbal, mathematical, diagrammatic, and graphical, through which concepts and problems can be expressed and communicated (Gao et al., 2022; Treagust et al., 2017). So, being skilled in evaluating and utilizing these representations has a great value in understanding the concepts and solving problems in physics (Opfermann et al., 2017; Theasy et al., 2018). This study presents the student's conceptual understanding of reflection and refraction of light. An indicator of the knowledge of the concepts in optics is the ability of students to comprehend and solve problems in different representational formats. Hence, a student who has developed a complete understanding of the topic can solve a problem in three other representations such as verbal, mathematical, and ray diagram.

Light reflection and refraction have been considered fundamental in studying physics, and previous studies have shown that teaching in the classroom concepts of light poses significant challenges. For instance, the students were not aware of the purpose of their ray diagrams. Although ray diagramming has the potential to help students have a better understanding and representation of the concept in geometric optics, students still often display difficulties in justifying their ray diagrams (Bancong et al., 2019; Heikkinen et al., 2016). One reason is that ray diagrams are presented in the textbooks only for solving straightforward problems of limited cases for image formation of mirrors and lenses (Heikkinen et al., 2016). Furthermore, a study on Thai students' difficulties in understanding the basic optics concepts such as refraction suggested that teaching students about ray diagrams need careful attention. It was reported that 1/220 high school student participants who had a qualitative understanding of how a ray diagram can be used to locate the position of an image of an actual object submerged in water could scientifically reason out how the image's correct position be identified (Kaewkhong et al., 2010).

A complete understanding of light reflection and refraction is attained when the students develop both conceptual and mathematical understanding of these topics (Ashmann et al., 2016). Also, students must illustrate ray diagrams to explain their knowledge of optics. However, students have difficulties learning optics conceptually, mathematically, and in terms of ray diagrams and the transfer between representations. Teaching light reflection and refraction in the classroom can be done conceptually, mathematically, and through ray diagrams, which are representations. Hence, teaching physics concepts with multiple representations to convey information can support students' construction of knowledge.

Multiple representations, which can be external as texts, graphs, pictures, or internal mental models, are used to deliver information and support the construction of knowledge (Kuo et al., 2017; Treagust et al., 2017). Learning with multiple representations indicates that two or more external representations are simultaneously utilized (Ainsworth, 2014; Cao et al., 2022). The use of multiple representations can support learning in three ways according to the DeFT (design, functions, and task) framework. First, students benefit from multiple external representations if each representation provides unique information or supports inferences (Ainsworth, 2014). It implies that various representations support understanding by containing qualitatively different aspects of the information to be learned or conveying similar information presented in different ways (Treagust et al., 2017). Another benefit is that multiple representations can also support learning if these representations limit each other's possible interpretations when presented simultaneously, especially when one representation is more familiar to the student than the other (Opfermann et al., 2017; Verwey et al., 2021). Finally, combining the multiple representations promotes a more profound understanding when students integrate information from different representation modes (Fatmaryanti et al., 2016); hence, students gain knowledge that is difficult to infer with only one representation. Due to the importance of multi-representation in learning, previous research has also focused on designing modules and e-books to improve students' conceptual understanding and representation skills (Mizayanti et al., 2020; Rasmawan, 2020; Resita & Ertikanto, 2018; Suarsana, 2021).

In physics education, representations are diverse forms in which physical concepts can be communicated, understood, and classified as verbal, mathematical, diagrammatic, and graphical. Studies have been on students' problem-solving performance and representations in physics education. Meltzer (2005) investigated the students' performance on similar Newton's third-law questions posted in verbal and diagrammatic problems. The results showed that students given two equal responded consistently higher in verbal representation than in diagrammatic representation and that the performance across representations is inconsistent. Moreover, Kohl and Finkelstein (2005) demonstrated that student performance varies across different representations of physics problems with similar contents, investigated why students perform differently on these representations, and showed that giving the students a choice of representation format will change their performance in problem-solving. Specifically, the student performance on physics problems varies enormously with representational formats between graphical and pictorial homework problems, better between pictorial and verbal formats of spectroscopy quizzes, but worse in three out of four subject areas when randomly assigned with the mathematical format of the problem.

Furthermore, De Cock (2012) examined the student success on three versions of a test item given in different representational formats: verbal, pictorial, and graphical, with an isomorphic problem statement that requires the same physics principle to solve them. The research results confirmed that the students' problem-solving competence could differ with representational formats. Specific details of the representations can generate solutions, and students use different problem-solving strategies depending on the representational format in which the problem is stated.

Multiple representations transform implicit knowledge explicitly, encourage students to solve problems with more than one approach, reorganize expertise, and represent a problem in numerous ways, just as a scientist does (Kohl & Finkelstein, 2008; Siswanto et al., 2018). Kuo et al. (2017) stated that it is much easier for students to integrate or translate different representations to understand the concepts better when using other representations and the meaning they represent. Moreover, multiple representations highlight aspects of the idea for students and lead to convergence across representations that may improve or strengthen the depth of understanding (Adadan, 2013).

Previous studies concerning multiple representations assert that if students can use different representations, it will deepen their understanding of the topic (Adadan, 2013; Kohl & Finkelstein, 2008; Kuo et al., 2017). However, looking from another point, students who are exposed to learning with multiple representations are expected to be able to recognize similar concepts presented in different representations. In other words, if students can use various representations to understand an idea, they can identify concepts of a topic in different representations. The complete understanding now is the sum of the parts of a concept shown in multiple representations. If the understanding is only observed in a representation, the students must have inconsistency or even difficulty learning the concepts of a topic in different representations. These formed ideas become the motivation of this study that seeks to determine the students' understanding of optics and their difficulties and inconsistencies in recognizing the concepts of the topic in multiple representations. Thus, this study investigates the recognition of the concepts in different representations and the inconsistencies and difficulties in understanding the concepts in various representations.

The primary purpose of this study is to determine whether students understand concepts of a reflection by mirrors and refraction by lenses presented in different representations such as verbal, mathematical, and ray diagrams. Moreover, to identify the students' inconsistencies and difficulties in understanding these topics. Specifically, to answer the following: (1) Do students understand the concepts of reflection by mirrors and refraction by lenses in three representations? (2) What inconsistencies do students have about the topic between the three representations? (3) What difficulties do students have with these phenomena in terms of these representations?

II. METHODS

This study chose two hundred thirty-one (N=231) Grade 10 high school students representing the top sections of two Filipino public high schools. The students (M=84, F=147) mostly had average and above-average knowledge of science and mathematics. They have studied light reflection by mirrors and refraction by lenses taught for 52 hours by two physics teachers with master's degrees in physics education in 2017. A prescribed module followed the high school general physics curriculum instructions that presented the topic in multiple ways. The concepts were generally taught verbally, while ray tracing and problem-solving activities were taught diagrammatically and mathematically.

This descriptive study applies the input-process-output model of research to analyze and describe the nature of the data (Creswell & Creswell, 2018). The research paradigm in Figure 1 summarizes the procedures taken in the study.

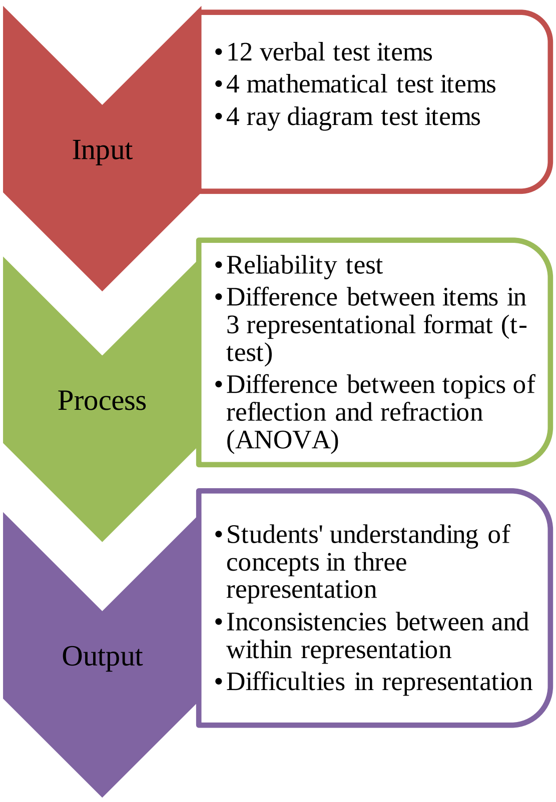
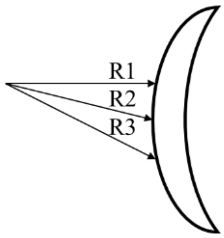
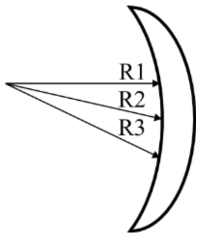
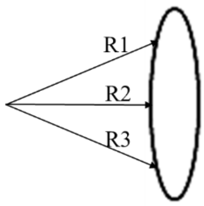
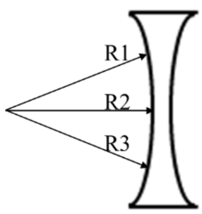


Figure 1. Research paradigm in this study

The test items, used initially to probe understanding of the concepts presented in the physics module, were administered to the participants after instructions. The problems are in the form of verbal and mathematical representations of light reflection and refraction with sub-topics on image formation and ray tracing. Twelve verbal items require conceptual analysis of the situation or background of the problem, while four (4) mathematical items involve computational evaluation of a given problem. These verbal and mathematical items are in the form of multiple-choice. Even though each representation requires students differently, test items presented in both representations can be understood through ray diagramming.

Four ray diagram items on spherical mirrors and lenses were added, requiring the students to apply their knowledge on light reflection and refraction for unconventional light rays reflecting from mirrors or refracting through lenses. Unconventional rays are not typical rays presented as given examples in the physics module. The three rays used in the test were as if coming from a point source reaching the mirror or the lens. The students' responses to the ray diagrams items were analyzed to explore the students' possible difficulties and inconsistencies. The test items were organized according to topic and sub-topics in verbal, mathematical, and ray diagram representation, as shown in Table 1.

Table 1. Verbal, mathematical, and ray diagram representation items

Topic			Representations					
			Item	**Verbal**	**Item**	**Mathematical**	**Item**	**Ray Diagram**
Reflection by Mirrors	Convex	*Image Formation*	1	What kind of mirror is used by department stores to give a wider area and smaller image of the shoppers/buyers?	4	An object is 6 cm in front of a convex mirror with a focal length of 6 cm. What is the location of the image formed?	17	
			2	Which of the following is/are true of a convex mirror? (I) It will never form an actual image. (II) An inverted image will be created if the object's distance is greater than the focal length. (III) An object can be magnified if it is placed at p=3f.				
		Ray Tracing	3	A light ray traveling obliquely to a convex mirror's axis goes directly to the mirror's center before striking the mirror's surface. What is the direction of the reflected ray after hitting the mirror?				
	Concave	*Image Formation*	5	An object is placed between a concave mirror and its focal point. What is the type and orientation of the image formed?	8	An object is kept 150 mm from a concave mirror with a radius of curvature of 600 mm. Find the image distance.	18	
			6	Where should an object be placed in front of a concave mirror so that the image will have the same size as the object?				
		Ray Tracing	7	A light ray traveling parallel to a concave mirror's axis strikes the mirror's surface. What is the direction of the reflected ray?				
Reflection by Lens	Convex	*Image Formation*	9	Light rays are observed to form at a point behind a lens. What kind of lens was used?	12	An object is kept at 80 cm from a convex lens of a focal length of 25 cm. Find the distance between its image and lens.	19	
			10	Sun's rays are observed to focus at a point behind the fishbowl near the window. The fishbowl act as what type of lens?				
		Ray Tracing	11	A light ray traveling parallel to the axis of a convex lens strikes the lens. What happens to this ray traveling through the lens?				
	Concave	*Image Formation*	13	What kind of image is formed by a convex lens?	16	An object 1.30 m tall is at 2.20 m from a concave lens. If the image is formed 15.0 cm from the mirror, what is the size of the mirror?	20	
			14	What type of lens produces smaller and upright images?				
		Ray Tracing	15	A light ray, traveling parallel to a concave lens' axis and strikes the lens, will refract and?				

Microsoft Excel was used to organize the test items' results and obtain the descriptive statistics such as frequency, mean, and standard deviation. In addition, the statistical statistics for the test were obtained by using SPSS Statistics. The t-test was used to determine any significant differences between the test item results by representations. The analysis of variances (ANOVA) was used to determine any significant differences between the topics of reflection by mirrors and refraction by lenses. The reliability of the test items was analyzed using the Cronbach Chi-square test of independence was also used to determine whether students answered things in one representation with their understanding of the concepts in another representation. The mean percentages were also obtained to describe the students' responses to ray diagram items and the correctness of the three rays in these drawings. Finally, these drawings were categorized according to the similar patterns observed.

The problems were administered to another group of Grade 10 students (N=165) to check their internal consistency. As the main participants of this study, these students have the same conditions on the time of instructions, knowledge of science and mathematics, type of curriculum, and teacher background. Using SPSS, the reliability statistics were found to have a Cronbach's alpha, α equal to .848, having the M=11.27 and SD=4.21. On the one hand, the same problems administered to the main participants (N=231) gave the results of M=10.73 and SD=3.41 and were found to have a proper internal consistency of α=.678. The results show that the problems were reliable and consistent in determining the students' understanding of light reflection by mirrors and refraction by lenses.

III. RESULTS AND DISCUSSION

This section begins with the students' understanding of the concepts understudy in the three representations, followed by the inconsistencies found between similar items in verbal and ray diagram representations, and finishes with the difficulty found.

Students' understanding of concepts in different representations

Students were assumed to have fragments of understanding of a concept in different representations, verbal, mathematical, and ray diagrams in this study. This fact can be seen in the differences between the results of the test items in the three representations. The results in Table 2 show that students mainly recognize the concepts of a topic in verbal representation. Verbal representation items particularly probed the conceptual understanding of students. The *t*-test result for the verbal representation (M=62, SD=.47) was found to be significantly different from the ray diagram (M=25, SD=.43) representation and is given by $t(3)=13.5$, p=0.001. This result implies that students could recognize the topics of light reflection by mirrors and refraction by lenses with the significant percentage results of the verbal representations items. However, no significant difference was found between the results by topic, with an ANOVA result of $F(3,8)=0.36$, p=0.78.

Table 2. Means (%) of students' understanding in three representations (N=231)

Topic	Verbal			Mathematical			Ray Diagram		
	N of Items	M (%)	SD	N of Items	M (%)	SD	N of Items	M (%)	SD
Reflection by Mirrors									
Convex	3	72	.44	1	45	.50	1	34	.48
Concave	3	67	.46	1	79	.41	1	23	.42
Refraction by Lenses									
Convex	3	56	.49	1	32	.47	1	21	.41
Concave	3	52	.47	1	77	.42	1	21	.41
Average		***62***	***.47***		***58***	***.45***		***25***	***.43***

Test of significance at $\alpha=0.05$. $t(3)=13.5$, p=0.001. $F(3,8)=0.36$, p=0.78

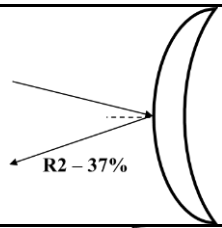
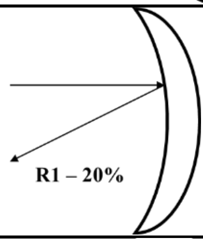
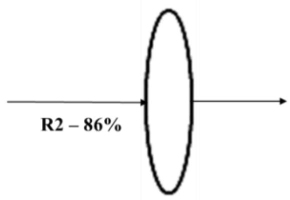
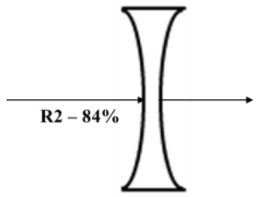
In terms of verbal representation, the reflection by convex and concave mirrors have higher mean percentages of 72% and 67%, respectively, compared to the refraction by convex and concave lenses with 56% and 53%, respectively. The reflection by a concave mirror and refraction by a concave lens have the higher percentages of 79% and 77% in terms of mathematical representation. In comparison, the reflection by the convex mirror and refraction by the convex lens have ratios of 45% and 32%, respectively. Lastly, the items garnered less than 50% in ray diagram representation.

Based on the results, the students seem to understand the concepts of the light reflected by mirrors rather than the refraction of lenses in verbal representation. Except for the mathematical problem on refraction by a concave lens, the remaining mathematical issues required the students to the distance of the image, d_i. They allowed the students to use the same mirror or lens equation. However, the results show that students considerably understand well the problem of the concave mirror in mathematical representation. The low percentages for the reflection by a convex mirror and the refraction by a convex lens suggest that students have inconsistent mathematical representation regarding using the mirror or lens equation. Finally, the considerably low percentages for the ray diagrams problems imply that students displayed difficulty representing the ray diagrams shown in their incorrect drawings of the three unconventional rays reflecting from mirrors or refracting through the lenses.

Inconsistencies between and within the representations

Inconsistencies between verbal and ray diagram representations were found. Inconsistencies were first explored between some items represented in verbal and ray diagrams. The verbal representation items 3, 7, 11, and 15 were compared with their corresponding rays from the ray diagrams items to analyze students' inconsistencies. These four items ask about the direction a light ray will take after reflecting from a mirror or refracting from a lens. Table 3 shows the discrepancies between the four items represented in the verbal and ray diagram representation. The three rays in the drawings are labeled R1, R2, and R3. The results show that students have different responses for items represented in both verbal and ray diagrams, indicating some inconsistencies.

Table 3. Inconsistencies in students' understanding of some items in verbal and ray diagram representations

Optical System	**Verbal Representation Items**	**Ray Diagram**
Convex Mirror (CXM)	Item 3 (**62%**) A light ray traveling obliquely to a convex mirror's axis goes directly to the mirror's center and strikes the mirror's surface. What is the direction of the reflected ray hitting the mirror?	
Concave Mirror (CVM)	Item 7 (**68%**) A light ray traveling parallel to a concave mirror's axis strikes the mirror's surface. What is the direction of the reflected ray?	
Convex Lens (CXL)	Item 11 (**60%**) A light ray, traveling parallel to the axis of a convex lens, strikes the lens. What happens to this ray after traveling through the lens?	
Concave Lens (CVL)	Item 15 (**52%**) A light ray, traveling parallel to a concave lens' axis and strikes the lens, will refract and?	

Item 3 is a verbal representation problem about how a light ray obliquely traveling toward a convex mirror will reflect after striking the mirror's center. This ray (R2) will be reflected at the same angle as it hits the mirror, following the law of reflection. Sixty-two (62) percent correctly answered in verbal representation, but only 37% correctly answered in the ray diagram representation. Item 7 is also a verbal representation problem about how a light ray parallel to a concave mirror's axis will travel after striking the mirror. This ray (R1) is expected to go through the focal point of a concave mirror. The mean percentages of correct responses are 68% in verbal representation and 20% in ray diagram representation.

Verbal representation item 11 is about which direction a light ray traveling parallel to the axis of a convex lens will be directed after refraction and can be represented in the diagram by the light ray (R2), which conventionally passes through the lens. Surprisingly, 89% of the total participants have correctly represented this ray in their ray diagrams, and 60% correctly answered in the verbal representation. Finally, verbal representation item 15, which is about how a light ray traveling parallel to a concave lens' axis will travel after refraction, is represented by a light ray (R2) in the ray diagram. Like the other first three items, there is a difference between the students' responses, 84% in the ray diagram and 52% in the verbal representation.

For all these items, students were found to have difficulty and could not draw the corresponding rays for convex mirror (12%), concave mirror (14%), convex lens (11%), and concave lens (10%). Given these related problems about light rays in verbal and ray diagram representations, it was found that students could not recognize the similar concepts represented in verbal and ray diagram representations. This result leads to the idea of identifying discrepancies in students' responses shown in Table 4.

The discrepancies between the students' responses in verbal and diagrams indicate inconsistency in their understanding of the concept probed in the problem. The percentage discrepancies (OX/XO) in the students' responses for the four represented items are bold in Table 4 On the other hand, the percentages of students who displayed correctness and consistency are 28%, 19%, 86%, and 42% for items 3, 7, 11, and 15, respectively.

Table 4. Discrepancy (%) in the correctness of students' responses for items in verbal and ray diagram (N=231, α=0.05).

Item 3*		**R**		**Item 7***		**R**		**Item 11****		**R**		**Item 15****		**R**	
		O	*X*			*O*	*X*			*O*	*X*			*O*	*X*
V	*O*	28	**24**	**V**	*O*	19	**35**	**V**	*O*	86	**3**	**V**	*O*	42	**3**
	X	**9**	28		*X*	**1**	32		*X*	**39**	.4		*X*	**41**	3

*p<0.05, **p>0.05. V - verbal, R - ray diagram, O - correct, X - incorrect. Discrepancies are written in bold.

A Chi-square test of independence was run through SPSS Statistics to determine if the students' responses in either verbal or ray diagram were independent of each other. It was found that the students' responses for verbal items 3 and 7 are associated with their drawings of rays R2 and R1 in their ray diagram representations given by $\chi^2(1) = 18.73$, p=.000 and $\chi^2(1) = 27.36$, p=.000 respectively. The result of the statistical analysis implies that students provided their ray diagrams for a convex mirror in item 3 and a concave mirror in item 7 based on their equivalent understanding of these optical systems in verbal representation. Hence, the discrepancies between these representations indicate that students failed to recognize the similar concepts of image formation in verbal and ray diagram representations. No independence was found between the students' responses for items 11 and 15.

Inconsistencies within the mathematical representations were also explored, particularly in mathematical representation. Items 4, 8, and 12 involved using the mirror or lens equation and required to find the distance of the image, di. Students were expected to apply this mathematical formula in answering the problem, but the results were 45%, 79%, and 32% for items 4, 8, and 12. Hence, the difference in the results indicates an inconsistency with using the mathematical equation during problem-solving.

A chi-square test of independence was again utilized to determine whether the students' answers for the three items were associated or not. The association would mean the same knowledge of the mathematical equation. For instance, item 8, having the highest mean percentage, was the basis of this association. Table 5 shows the discrepancies in the students' responses between the three items. It was found that the student's responses for mathematical items 4 and 12 are associated with their response for item 8. Given the results of $\chi2(1) = 32.94$, p=.000 and $\chi2(1) = 15.17$, p=.000, the assumption that students used the same knowledge of the mirror and lens equation to answer the three mathematical items seems to be true. Hence, it implies that students exhibited inconsistency with the use of the equation during the problem-solving on image formation.

Table 5. Discrepancy (%) in the correctness of students' responses for items in verbal, mathematical representation (N=231, α=0.05).

		Item 4*		**Item 8***		**Item 12***	
		O	*X*	*O*	*X*	*O*	*X*
Item 4	*O*			43	**2**	19	**26**
	X			**36**	19	**13**	42
Item 8	*O*	43	**36**			30	**49**
	X	**2**	19			**2**	19
Item 12	*O*	19	**13**	30	**2**		
	X	**26**	42	**49**	19		

*p<0.05, O - correct, X - incorrect. Discrepancies are written in bold.

Difficulties in mathematical and ray diagram representations

The results of the items in different representations are shown in Table 6. Verbal representation items have high percentages of results except for items 9 (42%) and 13 (30%). This result supports the earlier findings that students recognize the concepts of image formation mostly in this representation. Regarding mathematical representation, items 4 and 12 have low percentages of 45% and 32%, indicating difficulty in problem-solving on image formation by a convex mirror and a convex lens. Particularly, students showed difficulty with these items, which are about using mirror or lens equation, $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. Given the values of either focal length, f, object distance, do, or image distance, di, the students solved for the unknowns. Item 4 and 12 required the value of the image distance. The low percentage results for these items suggest that although students were given the derived forms of the equation and memorized these equations in the class, they still could not apply these equations and had difficulty solving problems on image formation by the spherical mirrors and the convex lens.

To further investigate this practical difficulty, mathematical representation items 8 and 16 integral equations $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ and $R = 2f$ and unit conversion were considered. The students' responses for items 8 and 16 are 79, and 77%, respectively. These problems should have been more difficult for the students since they needed to derive and apply other equations, but surprisingly their responses were contrary to what was expected. This gives the idea that students have difficulty using the mirror or lens equation.

Table 6. Item results per representation (N=231)

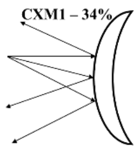
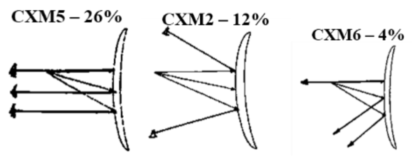
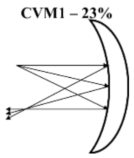
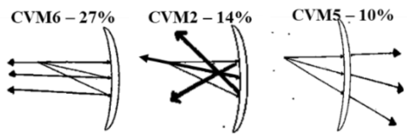
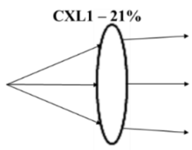
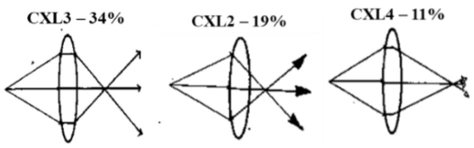
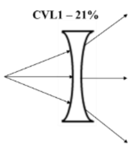
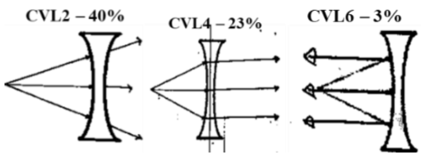
Topic		Sub-topic	Verbal			Representations Mathematical			Ray Diagram		
			Item	M (%)	SD	Item	M (%)	SD	Item	M (%)	SD
Refraction by Mirrors	Convex	*Image Formation*	1	79	.40	4	**45**	.50	17	**34**	.48
			2	74	.44						
		Ray Tracing	3	62	.49						
	Concave	*Image Formation*	5	59	.49	8	79	0.41	18	**23**	.42
			6	76	.43						
		Ray Tracing	7	68	.47						
Refraction by Lenses	Convex	*Image Formation*	9	**42**	.49	12	**32**	.47	19	**21**	.41
			10	66	.48						
		Ray Tracing	11	60	.49						
	Concave	*Image Formation*	13	**30**	.46	16	77	.42	20	**21**	0.41
			14	74	.44						
		Ray Tracing	15	52	.50						

Low percentages are written in bold.

Finally, the low percentages garnered by the ray diagram items suggest that, in general, students have difficulty in drawing the image formation by the spherical mirrors and lenses. Further analysis and categorization of the students' drawings were done based on the similarity of their features. The students' drawings were labeled CXM for a convex mirror, CVM for a concave mirror, CXL for a convex lens, and CVL for a concave lens. The categories were also numbered according to most to least frequently drawn ray diagrams. The drawing types for a convex mirror, for example, are CXM1, CXM2, CXM3, and so forth. The correct ray diagram and the top three most drawn alternative ray diagram drawings for image formation by the spherical mirrors and lenses are shown in Table 7.

The correct ray diagram for a convex mirror is CXM1 (34%), with the following alternative diagrams of CXM5 (26%) showing that light rays reflect parallel to each after hitting the convex mirror, and CXM6 (4%) indicating that R1 reflects parallel while R2 and R3 follow the law of reflection. Another alternative, CXM2 (12%), shows a light ray R2 not reflecting from the convex mirror.

Table 7. Alternative ray diagram representation for image formation by mirrors and lenses. (N=231)

Optical System	Ray Diagram: Correct	Ray Diagram: Alternatives
Convex Mirror (CXM)		 **No Drawing - 12%**
Concave Mirror (CVM)		 **No Drawing - 14%**
Convex Lens (CXL)		 **No Drawing - 8%**
Concave Lens (CVL)		 **No Drawing – 8%**

Twenty-three (23) percent of the total participants correctly drew the ray diagram of a concave mirror. Ten (10) percent thought that a concave mirror refracts rays. The alternative ray diagrams show that the three rays striking a concave mirror will reflect parallel (CVM6, 27%) or converge at a point in front of the mirror (CVM2, 14%).

In terms of the convex lens, 21% correctly drawn the ray diagram and the alternative ray diagrams show that light rays refract and converge in front (CXL, 19%), somewhat closer (CXL3, 34%), and far away (CXL4, 11%) from the convex lens. Students were not aware of the dependence of convergence on the incidence of the rays.

Finally, the correct ray diagram for a concave lens also garnered 21%. In comparison, the alternative diagram CVL2 (40%) shows that light rays will just go through the lens without refraction, CVL4 (23%) indicates that light rays are refracted parallel to the axis, and CVL6 (3%) shows that light rays are reflected from this refracting lens.

Even though students knew the functions of the optical systems and learned the laws of reflection and refraction and the rules of ray diagramming in the class, they could not apply this knowledge to provide the correct draw the ray diagrams of these optical systems in the test. Based on the drawings, less than 50% of the total participants could draw the accurate ray diagrams for the four optical systems, implying that students have difficulty removing the ray diagram of image formation by mirrors and lenses. Specifically, they have confusion between mirrors, lenses, and their respective types and could not apply the laws of reflection and refraction in their illustrations. Due to these difficulties, students failed to evaluate how the light rays would be reflected or refracted and just drew what they had memorized from the module.

This study aimed to determine the inconsistencies and difficulties Grade 10 students have in understanding reflection by mirrors and refraction by lenses in multiple representations such as verbal, mathematical, and ray diagrams. The results of this investigation can be summarized as follows:

Students could understand the concepts of image formation better in verbal representation than in mathematical and ray diagram representation, supported by the results. Notably, the results showed high percentages in verbal representation items on reflection by mirrors compared to refraction by a lens. The mean percentage in verbal was significantly different from the mean rate in the ray diagram representation items. The finding suggests that most of the students have a better understanding of concepts in verbal representation.

The general findings are similar to the previous research on geometric optics (Özdemir et al., 2020; Tural, 2015). Tural (2015) determined the students' conceptual understanding levels at different education levels relating to lenses. He found that students at all levels, primary, tertiary, and graduate education, lack knowledge and experience conceptual problems about lenses despite learning the concept in school. The study of Özdemir et al. (2020) on the conceptual understanding levels of opticians showed that the professionals had a low level of understanding with several misconceptions and insufficiencies related to the light and optics concepts. They implied that physics education for opticians is of great importance.

It was found that inconsistencies exist between items both represented in verbal (V) and ray diagram (R) representations, shown by the differences in the results of the test items involving light rays reflecting from spherical mirrors and refracting from the spherical lens. These inconsistencies are also supported by the discrepancies in the number of students who correctly answered these items in verbal and ray diagram representations. The inconsistencies suggest that students could not apply their understanding of the concepts of image formation in different contexts and representations. Inconsistencies within representations were also explored by comparing the results of the mathematical representations that involved using the mirror or lens equation and required to find the distance of the image, d_i. The difference was assumed to indicate inconsistency with the help of the mathematical equation during problem-solving. So, using the Chi-square test of independence, the association of the answers to the three items showed that the assumption that students used the same knowledge of the mirror and lens equation to answer the three mathematical items holds true. Thus, it implies that students exhibited inconsistency with the use of the equation during the problem-solving on image formation.

This finding of the study may be a basis for further exploration of the work of Şengören (2014) on the use of multiple representations for solving image formation problems. In his work, prospective physics teachers who used problem pictures, mirror/lens, and ray diagrams were more successful in problem-solving. Our study implies that having consistency in utilizing the three representations in the problem should be considered to conclude that a student is successful in problem-solving. The alternative ideas about using multiple representations in the study of Şengören (2014) may indicate inconsistency. Hence, it needs further investigation.

Furthermore, it was identified that students have difficulty having the topic under study in terms of mathematical and ray diagrams representations. The low percentage of items on image formation by a convex mirror and convex lens indicate that students have difficulty with image formation by mirrors and lenses in mathematical representation. Although students have memorized mirror and lens equations, they could not still apply these equations and had difficulty solving problems in the test requiring these equations. The students' alternative drawings of ray diagrams indicate their plight in this representation of image formation by mirrors and lenses. The results suggest that even students who learned the laws of reflection and refraction and the rules of the ray-tracing method still could not apply this knowledge in their drawings. Students also displayed confusion with mirrors and lenses and have drawn those light rays will (i) be reflected from a concave lens, 3%, and (ii) be refracted through a concave mirror, 10%.

IV. CONCLUSION AND SUGGESTION

In conclusion, this study unveils the teaching and learning reflection and refraction problem, specifically image formation. Inconsistencies and difficulties observed in this study point toward the students' incomplete understanding of the topic. The students' poor and incomplete understanding of a topic is challenging to correct and leads to various inconsistencies and difficulties determined in this study. Nonetheless, conceptual knowledge can become meaningful only when it is utilized to explain new situations, which can be done by presenting a specific topic in multiple representations. Thus, physics educators would probably agree that a complete understanding of a topic depends on three representations: conceptual knowledge (verbal) and mathematical understanding (mathematical), and correct drawing of light rays reflecting from mirrors or refracting through lenses (ray diagram).

Further investigation of the use of multiple representations to address this incomplete understanding of image formation is the primary recommendation for the problem. Specifically, a study on verbal representation problems can be solved by applying mathematical formulae and ray diagrams to explain further how students understand the topic in multiple representations.

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