JPF | Volume 14 | Number 2 | 2026 | 295 - 316

p - ISSN: 2302-8939
e - ISSN: 2527-4015

Jurnal Pendidikan Fisika

https://journal.unismuh.ac.id/index.php/jpf

DOI: 10.26618/0jeqap46

Application of Gauss-Jordan in Fixed Pulley System Solving to Mathematical Thinking Abilities and Learning Outcomes

Maryani1)*, Bambang Supriadi1), Ummi Zahrotul Ainiyah1), Aulia Wulansari Agustin1), Lutfiah Hafifatul Jannah1), Sakti Kalisa Sefanda1), Shinta Nuriyah Mahbubiyah Royani2)

1) Department of Physics Education, Universitas Jember, Jember, 68121, Indonesia
2) Department of Physics Education, Universitas Negeri Malang, Malang, 65145, Indonesia

**Corresponding author: maryani.fkip@unej.ac.id*

Received: December 22, 2025; Accepted: March 17, 2026; Published: May 04, 2026

Abstract *- Mathematical thinking ability is an essential component of physics learning because many physical phenomena are represented through mathematical models and systems of equations. However, students often have difficulty solving fixed-pulley system problems because they must simultaneously interpret physical concepts and solve systems of linear equations with multiple variables. This study aimed to analyze the application of the Gauss-Jordan elimination method in learning fixed pulley systems and to examine its effect on mathematical thinking ability, learning outcomes, and student responses. The study employed a quantitative approach with a posttest-only control-group quasi-experimental design. The participants comprised 130 respondents across two educational levels: 61 university students from the Physics Education Study Program at the University of Jember and 69 eleventh-grade students from SMAN 3 Jember. The experimental class was taught using the Gauss-Jordan elimination method, while the control class was taught using the substitution-elimination method. Data were collected through essay tests, learning outcome tests, and response questionnaires, and analyzed using the Independent Samples t-test and the Mann-Whitney U test, depending on the data distribution. The results showed that among school students, significant differences were found in the indicators of specializing (0.016), generalizing (0.001), and conjecturing (0.031), but not in convincing (0.468). Among university students, significant differences were found across all mathematical thinking indicators: specializing (0.007), generalizing (0.000), conjecturing (0.018), and convincing (0.004). Learning outcomes also differed significantly for both university students (0.000) and school students (0.010). In addition, student responses to the method's implementation were positive, averaging 71%. The novelty of this study lies in the integration of the Gauss-Jordan elimination method as a structured mathematical procedure in solving fixed pulley system problems in physics learning across two educational levels. In conclusion, the Gauss-Jordan elimination method can serve as an alternative instructional strategy that supports mathematical thinking, improves learning outcomes, and strengthens the connection between physical concepts and mathematical representations, thereby contributing to more systematic and meaningful physics education.*

Keywords: *fixed pulley; Gauss-Jordan elimination; learning outcomes; mathematical thinking; physics education.*

I. INTRODUCTION

The demands of the 21st century require education to focus not only on content mastery but also on the development of higher-order thinking skills and learners' character. Accordingly, 21st-century learning is positioned as a strategic approach for fostering moral development, intellectual capacity, and essential competencies such as critical thinking, creativity, problem-solving, knowledge construction, and deep mastery of the subject matter (Syahputra, 2024). Mastery of these competencies is an important foundation for enhancing students' competitiveness, particularly in responding to the demands for high-quality human resources in the global era (Kamil, 2023). To achieve this, education needs to direct learning toward approaches that strengthen students' reasoning abilities by emphasizing not only final answers but also the thinking processes involved in learning (Armelia & Ismail, 2021). One of the essential skills aligned with these demands is mathematical thinking ability, which plays an important role in fostering systematic ways of thinking among students (Rizki & Priatna, 2019).

Mathematical thinking ability refers to the capacity to use reasoning to formulate arguments, design strategies or solution methods, understand content, and communicate ideas through mathematical approaches (Rahayu et al., 2022). This ability plays an important role in learning because it helps students understand problems and identify appropriate solutions (Shidqiya & Sukestiyarno, 2022). Activities involved in mathematical thinking include exploring, questioning, working systematically, visualizing, connecting concepts, explaining, generalizing, reasoning, and proving. Therefore, mathematical thinking reflects students' mental activity in solving problems and is inseparable from the learning process, substantially affecting their learning outcomes (Devlin, 2021).

Physics and mathematics are closely interconnected, as almost all concepts in physics are expressed through mathematical models and equations. Previous studies confirm that learning physics through mathematical modeling can improve students' understanding of physics concepts. According to Baran-Bulut and Yüksel (2023) and Patero (2023), incorporating mathematical modeling into physics instruction can significantly improve learning outcomes. Difficulties in understanding physics are often associated with weak mastery of the mathematical concepts and symbols embedded in physics formulas (Piramanayagam et al., 2024). This indicates that mathematical competence is not merely supportive but fundamental to comprehensively understanding and applying physics concepts.

One topic in physics that is often considered difficult is the application of Newton's laws in motion dynamics. Understanding Newton's law of motion dynamics requires students to first master the concepts of velocity and acceleration (Alarabi et al., 2022). Dharma et al. (2025) reported that students' mastery of fixed-pulley-system material remains low, as evidenced by numerous errors when solving related problems. Rohmawati et al. (2023) also found that students' learning outcomes on pulley system material were still at a moderate level. In addition, Mulyastuti et al. (2019) found that students' ability to complete mathematical procedures on fixed pulley system material reached only 35.2%. These findings indicate that the pulley system remains challenging for students and that more systematic problem-solving methods are needed in physics learning.

In many classroom practices, students tend to rely on direct formula substitution without fully understanding the mathematical structure of the equations derived from physical principles (Bowers et al., 2024; Zhao & Schuchardt, 2021). When analyzing fixed pulley systems, students must simultaneously consider several physical quantities, such as tension, acceleration, torque, and rotational motion. These relationships often produce a system of linear equations with multiple variables that students must solve to determine the correct physical quantities. However, many students have difficulty handling multiple equations simultaneously, leading to procedural errors and incorrect reasoning during problem solving (Suryanti et al., 2024; Wahab et al., 2024). As a result, students may understand the physical concepts at a conceptual level but still fail to translate them accurately into mathematical solutions.

The persistent difficulties in understanding pulley systems highlight the importance of applying appropriate problem-solving methods to improve physics learning outcomes. Students' responses in previous studies suggest that applying mathematical methods to solve pulley system problems is associated with a positive trend toward improved mathematical thinking ability and learning outcomes (Supriadi et al., 2025). One study by Ni'mah et al. (2025) showed that the use of mathematical methods, such as Cramer's rule, obtained a positive student response of 82.14% in solving physics problems involving systems of linear equations (LES). This suggests that structured mathematical methods can help students understand the problem-solving process more logically and systematically. Furthermore, Arefin (2021) stated that the Gauss elimination method is more effective in solving LES than the inverse matrix method. Therefore, mathematical methods can be applied as alternative approaches for solving physics problems expressed in the form of LES.

The analysis of motion in a fixed pulley system with mass involves physical quantities such as torque, moment of inertia, and angular acceleration (Escalona & Mohammadi, 2022). In a fixed pulley system, the forces acting on the system must also satisfy the principles of Newton's law (Oliveira & Lemos, 2018). The application of Newton's law in rotational motion is used to determine the relationship among torque, moment of inertia, and angular acceleration (Nugraha, 2019). By applying these principles to a fixed pulley system, a system of three linear equations is obtained.

$$k_1x + l_1y + m_1z = n_1 \tag{1}$$

$$k_2x + l_2y + m_2z = n_2 \tag{2}$$

$$k_3x + l_3y + m_3z = n_3 \tag{3}$$

In general, LES is commonly solved using the substitution-elimination method (Fadhilah et al., 2021). However, this method becomes less efficient for systems with more than two variables because it requires repeated substitutions and sequential elimination steps (Indriati, 2019). Rahma et al. (2020) also explained that solving multivariable linear equations using substitution methods often leads to calculation errors and longer procedures, especially when students must manipulate several equations simultaneously. Consequently, students may focus more on computational procedures than on understanding the relationships among physical quantities. Therefore, an alternative solution method that is more systematic and efficient for solving complex LES is needed. One matrix algebra method used to solve LES is Cramer's rule, which determines the solution through matrix determinants (Saquin & Ancog, 2025). Another alternative is the Gauss-Jordan elimination method, which applies elementary row operations (ERO) until the matrix reaches reduced row echelon form.

One of the matrix algebra methods for solving LES is Gauss-Jordan elimination. Batarius & Samane (2021) stated that the Gauss-Jordan elimination method is effective for solving LES with more than two variables. The method also provides a structured solution procedure for solving complex linear equations (Ni'mah et al., 2025). According to Singh (2021), the stages of solving LES using the Gauss-Jordan elimination method are as follows:

(1) Convert LES (equations 1, 2, 3) to matrix multiplication equations

$$A\varphi = D \tag{4}$$

$$\text{with } A = \begin{bmatrix} k_1 & l_1 & m_1 \\ k_2 & l_2 & m_2 \\ k_3 & l_3 & m_3 \end{bmatrix};\ \varphi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ and\ B = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

(2) Converting equation (4) into an Augmented matrix

$$\left[\begin{array}{ccc|c} k_1 & l_1 & m_1 & n_1 \\ k_2 & l_2 & m_2 & n_2 \\ k_3 & l_3 & m_3 & n_3 \end{array}\right] \tag{5}$$

(3) Through elementary line operations, convert the Augmented matrix into a reduced line echelon matrix

$$\left[\begin{array}{ccc|c} k_1 & l_1 & m_1 & n_1 \\ k_2 & l_2 & m_2 & n_2 \\ k_3 & l_3 & m_3 & n_3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_1 \\ 0 & 1 & 0 & r_2 \\ 0 & 0 & 1 & r_3 \end{array}\right] \tag{6}$$

(4) Concluding the results of the Gauss-Jordan elimination, namely:

$$x = r_1, y = r_2, \tag{7}$$

Although Gauss-Jordan elimination is widely used in mathematics and engineering, its application as a structured problem-solving strategy in physics education, particularly for solving fixed-pulley system problems, has received limited attention. Most physics instruction still emphasizes formula-based approaches rather than integrating systematic mathematical procedures to solve equations derived from physical models. This indicates a gap between mathematical methods that can support structured reasoning and their implementation in physics education.

Therefore, integrating the Gauss-Jordan elimination method into physics problem solving may help students understand both the mathematical structure of equations and the physical relationships involved in pulley systems. By applying this method, students are expected to develop more systematic mathematical thinking while solving physics problems involving multiple variables. Accordingly, this study aims to analyze the application of the Gauss-Jordan elimination method in learning fixed pulley systems and to examine its influence on students' mathematical thinking ability, learning outcomes, and learning responses.

II. METHODS

This study employed a quantitative approach with a posttest-only control-group quasi-experimental design. This design was selected to examine the application of the Gauss-Jordan elimination method in the experimental class and the substitution-elimination method in the control class. The study involved participants from two educational levels, with a total sample of 130 respondents: 61 students from the Physics Education Study Program at the University of Jember and 69 eleventh-grade students from SMAN 3 Jember. Sampling was conducted using a random sampling technique to ensure that the control and experimental classes had equivalent initial ability characteristics. The Gauss-Jordan elimination method study procedure, as illustrated in Figure 1.

Figure 1. The Gauss-Jordan elimination method study procedure

During the implementation phase, the Gauss-Jordan method was applied in the experimental class through active-learning interactions focused on strengthening understanding of mathematical algorithms in a physics context. Participants were guided to model the physical forces in a fixed pulley system using Newton's Second Law to formulate a system of linear equations, which was then transformed into an augmented matrix. Classroom activities were conducted through guided exercises in which students independently practiced elementary row operations (ERO) on their worksheets. This process was intended to develop structured and precise logical thinking, thereby helping learners manage the complexity of variables in pulley systems, which are often difficult to solve using the substitution-elimination method.

The instrument used to measure mathematical thinking ability was an essay test developed based on four key indicators proposed by Mason, namely: (1) specializing, (2) generalizing, (3) conjecturing, and (4) convincing (Stacey, 2006). The test results and questionnaire responses were analyzed using descriptive and inferential statistics. Prior to hypothesis testing, prerequisite tests were conducted, including the Shapiro-Wilk normality test and the homogeneity-of-variance test. Because the data distribution varied, this study employed two statistical approaches. The Independent Samples t-test was used for normally distributed data, whereas the Mann-Whitney U test was used for non-normally distributed data as a non-parametric alternative. The Mann-Whitney U test was used due to its robustness to skewed data distributions, enabling statistically valid and accurate comparisons between groups to draw research conclusions.

In addition to examining the effect of the Gauss-Jordan elimination method on mathematical thinking ability and learning outcomes, this study administered a questionnaire on the application of the Gauss-Jordan elimination method to rotational dynamics problems. The questionnaire consisted of five indicators: interest, motivation, engagement, satisfaction, and feedback (Kartini & Putra, 2020). The criteria used to score the responses are presented in Table 1.

Table 1. Response outcome criteria

Criteria	Score
Strongly agree	5
Agree	4
Undecided	3
Disagree	2
Strongly disagree	1

The results of the student response questionnaire were converted into percentages using Equation (8) and then classified according to the response percentage categories shown in Table 2.

$$\text{Percentage} = \frac{\text{total score obtained}}{\text{maximum score}} \times 100\% \qquad (8)$$ (Ilyas & Liu, 2019)

Table 2. Percentage categories of response results

Percentage	Category
$80\% < P \leq 100\%$	Very positive
$60\% < P \leq 80\%$	Positive
$40\% < P \leq 60\%$	Moderately positive
$20\% < P \leq 40\%$	Somewhat positive
$0\% < P \leq 20\%$	Not positive

Source: (Herliana et al., 2025)

III. RESULTS

To solve problems involving a fixed pulley system, Newton's second law for translational motion, $\sum F = ma$, and the rotational dynamics equation, $\sum \tau = I\alpha$, were applied. As an example, two blocks with masses $m_1 = 6$kgs and $m_2 = 3$kgs are connected by a massless string passing over a fixed pulley. The pulley is assumed to be a solid thin cylinder with a mass of $M = 2$kg, and the gravitational acceleration is $g = 10$ m/s^2. To determine the objects' accelerations and the rope's tensions, a free-body diagram was first constructed, as shown in Figure 3.

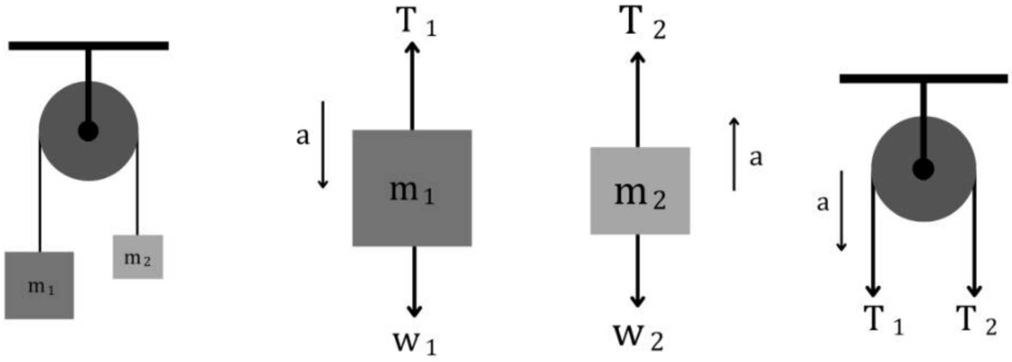


Figure 3. Free-body diagram

Based on the free-body diagram, the following system of linear equations (LES) was obtained:

i. Blocks 1:$T_1 + 6a = 60$ (9)
ii. Blocks 2: $T_2 - 3a = 30$ (10)
iii. Pulley: $T_1 - T_2 - a = 0$ (11)

The LES in Equations (9), (10), and (11) was solved using the Gauss-Jordan elimination method through the following stages.

a. The LES was converted into a matrix multiplication equation $A\varphi = D$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ a \end{bmatrix} = \begin{bmatrix} 60 \\ 30 \\ 0 \end{bmatrix} \tag{12}$$

with $A = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 1 & -1 & -1 \end{bmatrix}, \varphi = \begin{bmatrix} T_1 \\ T_2 \\ a \end{bmatrix}$, and $D = \begin{bmatrix} 60 \\ 30 \\ 0 \end{bmatrix}$

b. Equation (12) was written in the form of an augmented matrix:.

$$\left[\begin{array}{ccc|c} 1 & 0 & 6 & 60 \\ 0 & 1 & -3 & 30 \\ 1 & -1 & -1 & 0 \end{array}\right] \tag{13}$$

c. Through elementary row operations (ERO), the augmented matrix was transformed into a reduced row echelon matrix:

$$\left[\begin{array}{ccc|c}1 & 0 & 6 & 60\\ 0 & 1 & -3 & 30\\ 1 & -1 & -1 & 0\end{array}\right] \xrightarrow{b_3 - b_1} \left[\begin{array}{ccc|c}1 & 0 & 6 & 60\\ 0 & 1 & -3 & 30\\ 0 & -1 & -7 & -60\end{array}\right] \xrightarrow{b_3 + b_2} \left[\begin{array}{ccc|c}1 & 0 & 6 & 60\\ 0 & 1 & -3 & 30\\ 0 & 0 & -10 & -30\end{array}\right]$$

Multiply row 3 by $\left(-\frac{1}{10}\right)$ to obtain:

$$\left[\begin{array}{ccc|c}1 & 0 & 6 & 60\\ 0 & 1 & -3 & 30\\ 0 & 0 & 1 & 3\end{array}\right] \xrightarrow{b_2 + 3b_3} \left[\begin{array}{ccc|c}1 & 0 & 6 & 60\\ 0 & 1 & 0 & 39\\ 0 & 0 & 1 & 3\end{array}\right] \xrightarrow{b_1 - 6b_3} \left[\begin{array}{ccc|c}1 & 0 & 0 & 42\\ 0 & 1 & 0 & 39\\ 0 & 0 & 1 & 3\end{array}\right] \quad (14)$$

d. Using the definition in Equation (7), the solution is obtained as follows:

$$T_A = 42\ N, T_B = 39\ N, and\ a = 3\ m/s^2 \quad (15)$$

Mathematical thinking ability

In this study, mathematical thinking ability was examined through four indicators: specializing, generalizing, conjecturing, and convincing. A summary of the mathematical thinking ability scores of school students and university students is presented in Table 3.

Table 3. Mathematical thinking ability scores

Indicator	Students				University students			
	Control class		Experiment class		Control class		Experiment class	
	Score	Mean	Score	Mean	Score	Mean	Score	Mean
Specializing	364	10.70	434	12.40	356	11.86	440	14.19
Generalizing	400	11.70	466	13.30	239	7.96	362	11.67
Conjecturing	354	10.40	433	12.40	188	6.26	263	8.48
Convincing	149	4.38	176	5.03	93	3.10	188	6.06

Based on Table 3, the mathematical thinking ability scores of students and university students in the control and experimental classes are presented for each indicator. Before analyzing the differences between the two classes, a normality test was conducted. The results showed that the mathematical thinking ability data for school students were not normally distributed, with a significance value of less than 0.05. In contrast, the data for university students were normally distributed, with a p-value greater than 0.05. Therefore, the Mann-Whitney U test was used for the student group, while the Independent Samples t-test was used for the university student group.

The results of the Mann-Whitney U test for each indicator of students' mathematical thinking ability are presented in Table 4.

Table 4. Mann-Whitney U test results for each indicator of students' mathematical thinking ability

Indicator	Mann-Whitney U	Wicolxon W	Z	Asymp. Sig. (2-tailed)
Specializing	397.500	992.500	-2.412	.016
Generalizing	326.000	921.000	-3.320	.001
Conjecturing	416.500	1011.500	-2.163	.031
Convincing	535.000	1130.000	-.726	.468

The results of the Independent Samples t-test for each indicator of university students' mathematical thinking ability are presented in Table 5.

Table 5. Independent samples t-test results for each indicator of university students' mathematical thinking ability

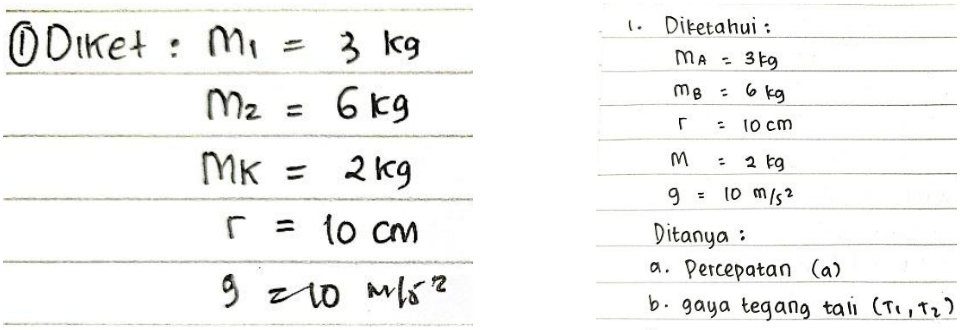
Indicator	t	df	Sig. (2-tailed)	Mean difference
Specializing	-2.835	39.544	.007	-2.327
Generalizing	-4.946	58.741	.000	-3.711
Conjecturing	-2.437	58.477	.018	-2.217
Convincing	-2.998	58.011	.004	-2.965

Based on Table 4, the indicators of specializing, generalizing, and conjecturing obtained Asymp. Sig. (2-tailed) values < 0.05, indicating significant differences between the control and experimental classes for these three indicators. However, the convincing indicator obtained an Asymp. Sig. (2-tailed) value greater than 0.05, indicating no significant difference between the two classes for this indicator. Based on Table 5, all four indicators had p-values < 0.05, indicating significant differences between the control and experimental classes for university students.

IV. DISCUSSION

Specializing

The specializing indicator refers to the ability of students and university students to understand information and analyze problems (Delima et al., 2021). The statistical results presented in Tables 4 and 5 indicate a significant difference in this indicator between the control and experimental classes. The students' work is shown in Figure 4.



① Diket : m_1 = 3 kg
m_2 = 6 kg
Mk = 2 kg
r = 10 cm
g = 10 m/s^2

1. Diketahui :
m_A = 3kg
m_B = 6 kg
r = 10 cm
M = 2 kg
g = 10 m/s^2
Ditanya :
a. Percepatan (a)
b. gaya tegang tali (T_1, T_2)

(a) Results of work on the control class by AR

(b) Results of work on the experimental class by WA

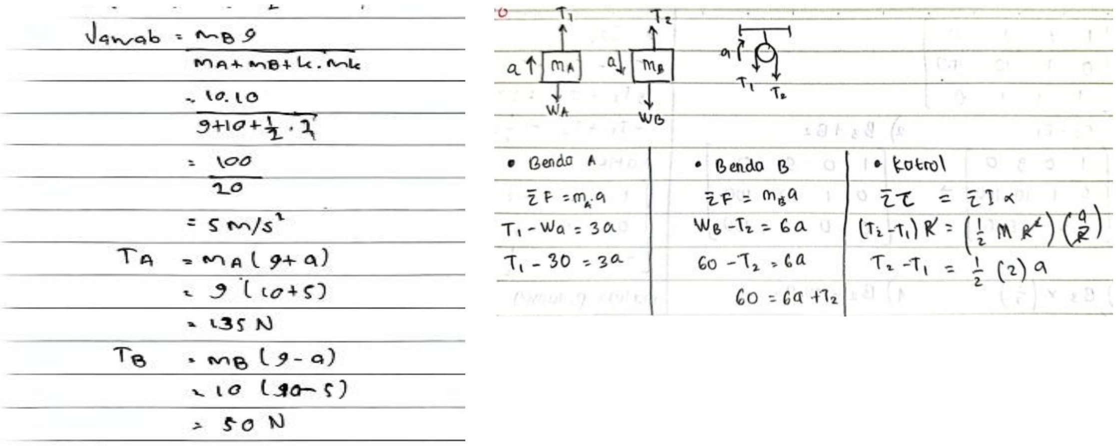
Figure 4. Results of the specializing indicator

Figure 4 shows a clear difference in problem-solving performance between the control and experimental classes. In part (a), students and university students in the control class wrote only part of the known and required information, which did not clearly identify what was being asked in the problem. This may be because they were not accustomed to writing the known and required information completely before beginning the solution process. In contrast, in part (b), students and university students in the experimental class wrote the known and required information more completely. Learners in the experimental class demonstrated greater awareness of the importance of including all relevant information to better understand the problem and to support a more systematic solution process in subsequent stages.

These findings have important implications for physics learning, particularly in problem solving. Clearly, writing down the known and required information is an essential initial step, as it helps students understand the problem's conditions before carrying out mathematical modeling. These findings are consistent with Haratua & Sirait (2016), who reported that students accustomed to writing known and required information before performing calculations tend to demonstrate a better understanding of the relationship between physics concepts and their mathematical representations.

Generalizing

The generalizing indicator concerns students' ability to transform problems into appropriate mathematical representations, such as free-body diagrams and systems of linear equations (Yenilmez et al., 2022). The statistical results presented in Tables 4 and 5 indicate a significant difference in the generalizing indicator between the control and experimental classes. The students' work is shown in Figure 5.



(a) Results of work on the control class by BPPK

(b) Results of work on the experimental class by MARP

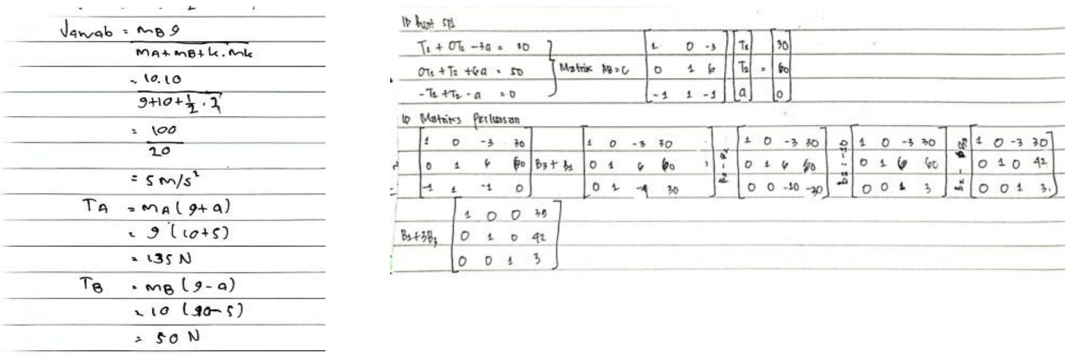
Figure 5. Results of the generalizing indicator

Based on Figure 5, there is a clear difference in the solution process between the control and experimental classes. In part (a), students and university students in the control class tended to omit the representation stage in the form of a free-body diagram or LES and directly applied the final formula. This indicates reliance on direct formula use without first constructing a mathematical model of the problem. In contrast, in part (b), students and university students in the experimental class were consistently guided to represent the problem mathematically by drawing the direction of forces in the free-body diagram and formulating an LES as the basis for the solution.

This difference indicates that using the Gauss-Jordan elimination method can help students generalize relationships among physical quantities into a more structured mathematical model before performing mathematical manipulations. These findings suggest that mathematical modeling in physics learning helps students understand the relationship between concepts and mathematical representations more deeply. The results are consistent with Maries and Singh (2023), who showed that using free-body diagrams and other visual representations when solving force-related problems can improve students' success by helping organize physical information more systematically. Similarly, Sirnoorkar et al. (2023) found that diagrammatic representations in physics problem solving can improve the quality of students' reasoning and problem-solving strategies. In addition, Sirait et al. (2025) emphasized that modeling and mathematical representation are essential components of developing physics problem-solving ability.

Conjecturing

The conjecturing indicator relates to students' ability to formulate the steps required to solve a problem (Yamin et al., 2022). The statistical results presented in Tables 4 and 5 indicate a significant difference in the conjecturing indicator between the control and experimental classes. The students' work is shown in Figure 6.



Jawab $= \frac{m_B g}{m_A + m_B + k.m_k}$

$= \frac{10.10}{9+10+\frac{1}{2}.2}$

$= \frac{100}{20}$

$= 5 m/s^2$

$T_A = m_A(g+a)$

$= 9(10+5)$

$= 135 N$

$T_B = m_B(g-a)$

$= 10(10-5)$

$= 50 N$

Buat SPL

$T_1 + 0T_2 - 3a = 30$

$0T_1 + T_2 + 6a = 50$

$-T_1 + T_2 - a = 0$

Matrix AB = C $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 6 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ a \end{bmatrix} = \begin{bmatrix} 30 \\ 60 \\ 0 \end{bmatrix}$

Matriks Perluasan

$$\begin{bmatrix} 1 & 0 & -3 & 30 \\ 0 & 1 & 6 & 60 \\ -1 & 1 & -1 & 0 \end{bmatrix} B_3 + B_1 \begin{bmatrix} 1 & 0 & -3 & 30 \\ 0 & 1 & 6 & 60 \\ 0 & 1 & -4 & 30 \end{bmatrix} B_3 - B_2 \begin{bmatrix} 1 & 0 & -3 & 30 \\ 0 & 1 & 6 & 60 \\ 0 & 0 & -10 & -30 \end{bmatrix} B_3 : -10 \begin{bmatrix} 1 & 0 & -3 & 30 \\ 0 & 1 & 6 & 60 \\ 0 & 0 & 1 & 3 \end{bmatrix} B_2 - 6B_3 \begin{bmatrix} 1 & 0 & -3 & 30 \\ 0 & 1 & 0 & 42 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$B_1 + 3B_3 \begin{bmatrix} 1 & 0 & 0 & 39 \\ 0 & 1 & 0 & 42 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

(a) Results of work on the control class by APNP

(b) Results of work on the experimental class by IFH

Figure 6. Results of the conjecturing indicator

Based on Figure 6, there is a noticeable difference in the solution process between the control and experimental classes. In part (a), students and university students in the control class tended to solve the problem by directly applying formulas. This approach causes learners to depend on memorized formulas, so when they encounter variations of pulley-system problems, they may lose direction if the memorized formula does not match the modified problem. In contrast, in part (b), students in the experimental class applied the Gauss-Jordan elimination method, which provides a consistent procedure for solving various problem types by transforming the system of equations into reduced row echelon form without requiring back substitution.

This finding is consistent with Rahma et al. (2020), who stated that the Gauss-Jordan elimination method allows the values of the variables to be determined directly, without the need for back substitution, thereby making the solution process more systematic and structured. Through this method, students first construct the mathematical relationships among forces as a system of linear equations, then perform elimination to determine the variables' values. This procedure helps them analyze relationships among variables in a more structured way, enabling them to carry out problem-solving steps logically and systematically. Furthermore, Musengimana et al. (2025) showed that applying systematic physics problem-solving strategies can help students organize solution steps in a structured manner, enabling them to plan the process more effectively and connect relevant physics concepts with appropriate procedures. These findings indicate that the Gauss-Jordan elimination method can support conjecturing ability by providing a clear, structured, and systematic framework for developing problem-solving strategies.

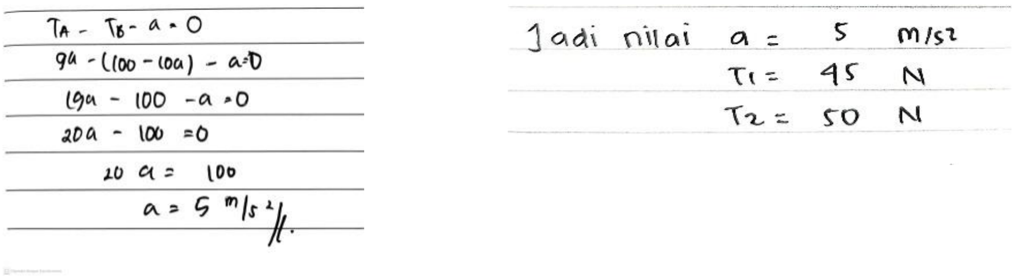
Convincing

The convincing indicator pertains to students' ability to present conclusions based on the final results of problem-solving (Hamidah & Suherman, 2016). The statistical results in Table 4 show that, at the school-student level, there is no significant difference between the control and experimental classes, whereas Table 5, at the university-student level, shows a significant difference. The students' work is presented in Figure 7.

Jadi nilai $a = 5\ m/s^2$, $T_A = 45\ N$, $T_B = 50\ N$

Jadi, nilai $a = 5\ m/s^2$
$T_1 = 45\ N$
$T_2 = 50\ N$

(a) Results of work on the control class by SS and the experimental class by VSAE



$T_A - T_B - a = 0$

$9a - (100 - 10a) - a = 0$

$19a - 100 - a = 0$

$20a - 100 = 0$

$20a = 100$

$a = 5\ m/s^2$ //.

Jadi nilai $a = 5\ m/s^2$

$T_1 = 45$ N

$T_2 = 50$ N

(b) Results of work on the control class by IFH and the experimental class by HN

Figure 7. Results of the convincing indicator

Based on Figure 7, part (a) shows that the work of school students in both the control and experimental classes does not differ substantially; students in both groups similarly wrote final conclusions based on their calculation results. This indicates that the ability to write conclusions had already emerged in both groups, so the treatment did not produce a significant effect on the convincing indicator. In contrast, part (b) shows a difference at the university-student level. University students in the control class tended to underline the final result without providing an explicit concluding statement, whereas those in the experimental class wrote the conclusion more completely.

In this study, the convincing indicator was the lowest among all mathematical thinking indicators because students and university students rarely wrote explicit conclusions. This condition suggests that convincing ability is influenced not only by mathematical solution procedures but also by learners' habits in communicating the results of their calculations. This finding is supported by Supriadi et al. (2025), who also found that the convincing indicator was the lowest component of mathematical thinking ability relative to the other indicators.

Overall, the results show significant differences in mathematical thinking ability between the control and experimental classes for both school students and university students. These findings indicate that learning supported by systematic and structured solution procedures plays an important role in developing mathematical thinking ability. By applying the Gauss-Jordan elimination method, students are directed to solve problems sequentially at each stage, allowing their mathematical thinking and accuracy to develop more effectively. This is in line with Midgett and Eddins (2001), which states that systematic and structured problem-solving strategies can develop higher-order thinking skills and strengthen connections between mathematical concepts. In addition, Kumari and Poonia (2021) stated that the Gauss-Jordan elimination method has a systematic solution procedure, thereby supporting mathematical thinking in a more directed manner. Similarly, Puspitasari et al. (2018) stated that strong mathematical thinking is reflected in fluency, flexibility, and originality in problem solving.

Learning outcomes

In addition to assessing mathematical thinking ability in fixed-pulley system problems, this study analyzed differences in learning outcomes between the Gauss-Jordan elimination method and the substitution-elimination method across educational levels. A summary of the learning outcome data is presented in Table 6.

Table 6. Learning outcome data

Group	Class	Sample size	Maximum score	Total score	Average
Students	Control	34	3400	2146.67	63.137
	Experimental	35	3500	2480.00	70.857
University students	Control	30	3000	1486.70	49.556
	Experimental	31	3100	2043.30	65.914

Based on Table 6, the average learning outcome scores in the experimental class were higher than those in the control class for both groups. This finding indicates a positive trend in the use of the matrix-based method to help students, including university students, organize solutions to physics problems. Before testing the hypothesis, a normality test was conducted using the Shapiro-Wilk test. The results showed that the university-student data were normally distributed, so the analysis proceeded with the parametric Independent Samples t-test, as shown in Table 7. In contrast, the school-student data were not normally distributed, so a non-parametric analysis was performed using the Mann-Whitney U test, shown in Table 8.

Table 7. Independent samples t-test of university students' learning outcomes

	t	df	Sig. (2-tailed)	Mean difference
Learning outcomes	-4.050	58.937	.000	-16.401

Based on the analysis in Table 7, a significance value of 0.000 (< 0.05) was obtained, indicating a significant difference in university students' learning outcomes between the two classes.

Table 8. Mann-Whitney U test of students' learning outcomes

	Mann-Whitney U	Wicolxon W	Z	Asymp. Sig. (2-tailed)
Learning outcomes	382.500	977.500	-2.570	.010

Based on the analysis in Table 8, a significance value of 0.010 (< 0.05) was obtained. This indicates a significant difference in students' learning outcomes between the two classes.

The results of the study indicate a significant difference in learning outcomes between the experimental and control classes for both school and university students. The higher average learning outcomes in the experimental class indicate that the Gauss-Jordan elimination method is more effective for solving problems involving fixed pulley systems. This difference suggests that a more systematic procedure for solving systems of linear equations can help students organize the mathematical model of physics problems more clearly. These findings are consistent with

Saraswati et al. (2020), who stated that foundational mathematical skills play a crucial role in physics learning outcomes. In addition, Badmus and Jita (2024) emphasized that difficulties in learning physics are often related to the ability to represent physics concepts in symbolic and mathematical forms. This is further supported by Fitriyana et al. (2025), who demonstrated that instruction providing structured guidance in the scientific process can significantly enhance students' conceptual understanding and physics learning outcomes. Thus, the Gauss-Jordan elimination method can guide the mathematical representation process, enabling students, including university students, to understand the relationships among variables in physics problems more clearly (Ni'mah et al., 2025).

The difference in learning outcomes between the two classes can also be explained by the characteristics of the methods used. The Gauss-Jordan elimination method provides a more structured solution process by reducing the matrix to a form from which the solution can be read directly, without further substitution. This procedural structure helps students, including university students, manage systems of linear equations with multiple variables more systematically. Kumari and Poonia (2021) stated that the Gauss-Jordan elimination method is efficient for solving complex systems of linear equations. This finding is also consistent with Dharma et al. (2025), which shows that using the Gauss-Jordan method to solve physics problems can help students, including university students, organize their solution steps more clearly.

In contrast, the substitution-elimination method requires a longer calculation process, thereby increasing the likelihood of algebraic errors as the number of variables increases (Supriadi et al., 2026). This indicates that structured mathematical procedures not only simplify calculations but also help students, including university students, trace the relationships among the physical variables in systems of linear equations. With a more systematic solution process, mathematical modeling for solving physics problems, particularly in pulley systems, can be carried out more effectively. This facilitates students' understanding of the connection between physical concepts and the mathematical representations used. These findings indicate that integrating the matrix-based Gauss-Jordan method into physics instruction has the potential to minimize procedural barriers in problem solving, thereby allowing learners' cognitive attention to focus more on understanding the underlying physics concepts.

Response

Student responses were obtained via a questionnaire administered after instruction on solving rotational dynamics problems using Gauss-Jordan elimination. Based on the analysis of the response data, the results for the five indicators are presented in Table 9.

Table 9. Response questionnaires result data

Indicator	Likert scale average percentage	Category	Category average
Interest	72%	Positive	71% positive
Motivation	72%	Positive	
Curiosity	66%	Positive	
Satisfaction	66%	Positive	
Feedback	72%	Positive	

Table 9 indicates that students' responses regarding the use of the Gauss-Jordan elimination method in solving fixed pulley system problems across five indicators averaged 71%, which falls into the positive category.

The interest indicator received a positive response of 72%, indicating that students were interested and enthusiastic about participating in the learning process and enjoyed solving fixed pulley system problems using the applied method. The motivation indicator also received a positive response of 72%, indicating that the application of the Gauss-Jordan method to solve fixed pulley problems increased students' motivation to learn. The curiosity and satisfaction indicators each scored 66%, indicating that students' understanding and appreciation of solving fixed-pulley system problems improved after using the method. The feedback indicator obtained 72%, indicating that students considered the problem-solving steps easy to understand and useful for providing additional insight into solving fixed pulley system problems.

The results show that applying the Gauss-Jordan elimination method to solve rotational dynamics problems yielded an average response score of 71%, categorized as positive. These positive responses indicate that applying structured, systematic solution steps can help students model fixed-pulley system problems as mathematical equations. Furthermore, this positive response aligns with the learning outcomes, in which the class applying this method demonstrated a better understanding. This is consistent with Dharma et al. (2025), who stated that the Gauss-Jordan elimination method is more effective in solving systems of linear equations. The Gauss-Jordan elimination method can therefore be used as an alternative approach for solving physics problems involving systems of linear equations. Thus, physics learning does not only focus on final answers, but also on students' ability to model mathematical equations and solve problems in a structured manner.

V. CONCLUSION AND SUGGESTION

The findings of this study show that applying the Gauss-Jordan elimination method to learning fixed pulley systems helps students, including university students, solve physics problems involving systems of linear equations in a more structured and systematic manner. The method significantly influenced students' mathematical thinking ability across the indicators of specializing, generalizing, and conjecturing, while at the university level it significantly affected all four indicators: specializing, generalizing, conjecturing, and convincing. In addition, the Gauss-Jordan elimination method produced significant differences in learning outcomes for both school students and university students. Students' responses to the method's implementation were also categorized as positive, indicating it was well received during the learning process.

This study is limited by its use of a posttest-only quasi-experimental design, the relatively small sample drawn from one university and one high school, and its focus on fixed-pulley system material only. Therefore, future research is recommended to include broader, more diverse samples, employ longitudinal or mixed-methods approaches, and examine the implementation of the Gauss-Jordan elimination method in other physics topics that involve mathematical modeling and systems of equations. Despite these limitations, this study contributes to the field of physics education by demonstrating that integrating a systematic, matrix-based solution method can strengthen mathematical thinking, improve learning outcomes, and help students connect physical concepts to mathematical representations more effectively.

REFERENCES

Alarabi, K., Tairab, H., Wardat, Y., Belbase, S., & Alabidi, S. (2022). Enhancing the learning of Newton's second law of motion using computer simulations. *Journal of Baltic Science Education, 21*(6), 946–966. https://doi.org/10.33225/jbse/22.21.946

Arefin, M. N. E. (2021). A unique approach to solve the system of linear equations. *International Journal of Innovative Technology and Interdisciplinary Sciences, 4*(1), 623–633. https://doi.org/10.15157/IJITIS.2021.4.1.623-633

Armelia, M. N., & Ismail, I. (2021). Pengaruh self-regulated learning terhadap kemampuan berpikir reflektif matematis siswa. *Jurnal Cendekia, 5*(2), 1757–1768. https://doi.org/10.31004/cendekia.v5i2.687

Badmus, O. T., & Jita, L. C. (2024). Physics difficulty and problem-solving: Exploring the role of mathematics and mathematical symbols. *Interdisciplinary Journal of Education Research, 6*, 1–14. https://doi.org/10.38140/ijer-2024.vol6.08

Baran-Bulut, D., & Yüksel, T. (2023). Interdisciplinary teaching: Solving real-life physics problems through mathematical modelling. *Electronic Journal for Research in Science & Mathematics Education, 27*(4), 118–140. https://ejrsme.icrsme.com/article/view/22761

Batarius, P., & Samane, I. P. A. N. (2021). Analisis metode Gauss-Jordan dalam penentuan arus pada rangkaian listrik. *Jurnal Ilmiah Matrik, 23*(3), 279–290. https://doi.org/10.33557/jurnalmatrik.v23i3.1508

Bowers, J., Anderson, M., & Beckhard, K. (2024). A mathematics educator walks into a physics class: Identifying math skills in students' physics problem-solving practices. *Journal for STEM Education Research, 7*(3), 335–361. https://doi.org/10.1007/s41979-023-00105-w

Delima, N., Kusumah, Y. S., & Fatimah, S. (2021). Students' mathematical thinking and comprehensive mathematics instruction (CMI) model. *Formatif: Jurnal Ilmiah Pendidikan MIPA, 11*(2), 161-172. https://doi.org/10.30998/formatif.v11i2.7807

Devlin, K. (2021). Teaching mathematics as a way of thinking—not calculating. *Eesti Haridusteaduste Ajakiri (Estonian Journal of Education), 9*(1), 33-59. https://doi.org/10.12697/eha.2021.9.1.02b

Dharma, N. D., Supriadi, B., Subiki, S., Maryani, M., & Putri, U. A. (2025). Implementasi eliminasi Gauss-Jordan dalam materi katrol tetap fisika kelas XI SMA/MA. *Eduproxima (Jurnal Ilmiah Pendidikan IPA), 7*(4), 2030–2038. https://doi.org/10.29100/.v7i4.8188

Escalona, J. L., & Mohammadi, N. (2022). Advances in the modeling and dynamic simulation of reeving systems using the arbitrary Lagrangian–Eulerian modal method. *Nonlinear Dynamics, 108*(4), 3985–4003. https://doi.org/10.1007/s11071-022-07357-y

Fadhilah, R., Sujadi, I., & Siswanto, S. (2021). The critical thinking process of senior high school students in problem-solving of linear equations system. *Journal of Physics: Conference Series, 1808*(1), 1-7. https://doi.org/10.1088/1742-6596/1808/1/012063

Fitriyana, S., Farhan, A., Susanna, S., Hamid, A., & Mahzum, E. (2025). The effect of guided inquiry learning model on students' science process skills and learning outcomes in physics science lessons. *Jurnal Pendidikan Fisika, 13*(3), 555–571. https://doi.org/10.26618/0m4ny850

Hamidah, K., & Suherman, S. (2016). Proses berpikir matematis siswa dalam menyelesaikanmasalah matematika ditinjau dari tipe kepribadian keirsey. *Al-Jabar: Journal of Mathematics Education, 7*(2), 231–248. https://doi.org/10.24042/ajpm.v7i2.38

Haratua, T. M. S., & Sirait, J. (2016). Representations based physics instruction to enhance students' problem solving. *American Journal of Educational Research, 4*(1), 1–4. https://pubs.sciepub.com/education/4/1/1/index.html

Herliana, F., Mardila, R., Mahzum, E., Zainuddin, Z., Wahyuni, A., Elisa, E., & Muliyati, D. (2025). The effect of web-based inquiry physics problems on high school students' physics learning outcomes. *Jurnal Pendidikan Fisika, 13*(2), 206–224. https://doi.org/10.26618/jpf.v13i2.17788

Ilyas, I., & Liu, A. N. A. L. (2019). Development of physics learning tools based on contextual teaching and learning in a remote island area. *Jurnal Pendidikan Fisika, 7*(1), 1–8. https://doi.org/10.26618/jpf.v7i1.1590

Indriati, K. (2019). *Matriks, vektor, dan program linier*. Unika Atma Jaya.

Kamil, F. (2023). Matematika sebagai fondasi kritis dalam menaklukkan tantangan soal fisika. *Journal of Educational and Applied Science, 1*(1), 1–6. https://doi.org/10.30739/jeas.v1i1.2486

Kartini, K. S., & Putra, I. N. T. A. (2020). Respon siswa terhadap pengembangan media pembelajaran interaktif berbasis android. *Jurnal Pendidikan Kimia Indonesia, 4*(1), 12–19. https://doi.org/10.23887/jpk.v4i1.24981

Kumari, K., & Poonia, R. K. (2021). A study of solving system of linear equation using different methods and its real life applications. *Journal of University of Shanghai for Science and Technology, 23*(7), 723–733. https://doi.org/10.51201/jusst/21/07197

Maries, A., & Singh, C. (2023). Helping students become proficient problem solvers part I: A brief review. *Education Sciences, 13*(2), 1-21. https://doi.org/10.3390/educsci13020156

Midgett, C. W., & Eddins, S. K. (2001). NCTM's principles and standards for school mathematics: Implications for administrators. *NASSP Bulletin, 85*(623), 35-42. https://doi.org/10.1177/019263650108562305

Mulyastuti, H., Sutopo, S., & Taufiq, A. (2019). Identification of high school students' problem-solving skills on rotational dynamics. *Journal of Physics: Conference Series, 1171*(1), 1-14. https://doi.org/10.1088/1742-6596/1171/1/012028

Musengimana, T., Yadav, L. L., Uwamahoro, J., & Nizeyimana, G. (2025). Effect of systematic physics problem-solving strategy on secondary school students' learning achievement. *Physics Education, 60*(3). https://doi.org/10.1088/1361-6552/adc8ed

Ni'mah, S., Supriadi, B., & Nuraini, L. (2025). Teaching material design for solving 2 & 3 loop electrical circuits using the Gauss-Jordan elimination method. *Journal of Teaching and Learning Physics, 10*(2), 110–120. https://doi.org/10.15575/jotalp.v10i2.44536

Nugraha, A. M. (2019). Graphic user interface (GUI) untuk materi dinamika gerak sistem katrol berbasis MATLAB. *Navigation Physics: Journal of Physics Education, 1*(2), 51–58. https://doi.org/10.30998/npjpe.v1i2.200

Oliveira, T. R. de, & Lemos, N. A. (2018). Force and torque of a string on a pulley. *American Journal of Physics, 86*(4), 275–279. https://doi.org/10.1119/1.5016040

Patero, J. L. (2023). The art of mathematical modeling in college physics: Strategies for fostering student engagement. *International Journal of Advanced Research in Science, Communication and Technology, 3*(1), 774–779. https://doi.org/10.48175/IJARSCT-12369

Piramanayagam, P., Bhalla, Y., Venkanna, K., Kumar, V. V., Sudheesh, P., & Sivaraman, R. (2024). Mathematical physics approaches to nanotechnology and material science. *Nanotechnology Perceptions, 20*(S16), 85–97. https://doi.org/10.62441/nano-ntp.vi.3614

Puspitasari, L., In'am, A., & Syaifuddin, M. (2018). Analysis of students' creative thinking in solving arithmetic problems. *International Electronic Journal of Mathematics Education, 14*(1), 49-60. https://doi.org/10.12973/iejme/3962

Rahayu, R., Bintoro, H. S., & Murti, A. C. (2022). The effect of self-confidence on the mathematical thinking ability of junior high school students. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika, 11*(4), 3826–3833. http://dx.doi.org/10.24127/ajpm.v11i4.5892

Rahma, A. N., Rahmawati, R., & Wahyuni, W. (2020). Metode eliminasi Gauss untuk penyelesaian sistem kongruensi linier. *Jurnal Sains Matematika dan Statistika, 6*(1), 30-39. https://doi.org/10.24014/jsms.v6i1.9250

Rizki, L. M., & Priatna, N. (2019). Mathematical literacy as the 21st century skill. *Journal of Physics: Conference Series, 1157*(4), 1-5. https://doi.org/10.1088/1742-6596/1157/4/042088

Rohmawati, Q., Siswanto, J., & Roshayanti, F. (2023). Kepraktisan dan efektivitas pembelajaran konsep dinamika rotasi berorientasi education for sustainable development (ESD) untuk meningkatkan keterampilan berpikir kreatif. *Jurnal Inovasi Pembelajaran di Sekolah, 4*(1), 193–200. https://doi.org/10.51874/jips.v4i1.75

Saquin, C., & Ancog, E. (2025). System of linear equations: A comparative study between Cramer's rule and Paravartya's rule. *Journal of Education and Learning Reviews, 2*(3), 1–20. https://doi.org/10.60027/jelr.2025.1303

Saraswati, D. L., Lestari, I., Seruni, S., Andinny, Y., & Hikmah, N. (2020). The effect of basic mathematical abilities on learning outcomes of physics education students. *Journal of Physics: Conference Series, 1464*, 1-4. https://doi.org/10.1088/1742-6596/1464/1/012008

Shidqiya, A. I., & Sukestiyarno, S. (2022). Analysis of students' mathematical thinking ability in terms of self-efficacy. *Unnes Journal of Mathematics Education, 11*(3), 272–281. https://doi.org/10.15294/ujme.v11i3.58772

Singh, A. (2021). *Introduction to matrix theory*. Springer. https://doi.org/10.1007/978-3-030-80481-7

Sirait, J., Oktavianty, E., Silitonga, H. T. M., & Ainley, J. (2025). Physics education students' views about force diagrams while solving physics problems. *Jurnal Pendidikan IPA Indonesia, 14*(2), 267-281. https://doi.org/10.15294/jpii.v14i2.21457

Sirnoorkar, A., Bergeron, P. D. O., & Laverty, J. T. (2023). Sensemaking and scientific modeling: Intertwined processes analyzed in the context of physics problem solving. *Physical Review Physics Education Research, 19*(1), 1-19. https://doi.org/10.1103/PhysRevPhysEducRes.19.010118

Stacey, K. (2006). What is mathematical thinking and why is it important?. *Proceedings of the APEC- Tsukuba International Conference 2007: Innovative Teaching Mathematics through Lesson Study*, 39–48. https://www.criced.tsukuba.ac.jp/math/apec/apec2007/paper_pdf/Kaye%20Stacey.pdf

Supriadi, B., Anggraeni, S. N. H., Purwanti, N. Y. N., Pujiningtiyas, E. B., Mahartika, D., & Wardhany, M. K. K. (2025). *Dinamika sistem katrol: Teori dan aplikasi matriks dalam gerak sistem katrol*. UPA Penerbitan Universitas Jember.

Supriadi, B., Arsita, M., Ni'mah, S., Wardhany, M. K. K., Afidah, Z., & Kinanti, A. Z. L. (2025). The effectiveness of Cramer's rule on the system of linear equations (LES) of 2-loop electrical circuits in improving mathematical thinking ability and learning outcomes. *Unnes Science Education Journal, 14*(1), 153–163. https://doi.org/10.15294/usej.v14i1.20515

Supriadi, B., Maryani, M., Putri, F. A., Zahro, R. F., Rohman, M. F., Sari, P. A. E., & Nafisah, N. (2026). The effect of algorithmic Gauss-Jordan method on mathematical reasoning and learning outcomes in dynamic electricity. *Jurnal Ilmiah Ilmu Terapan Universitas Jambi, 10*(1), 181–200. https://doi.org/10.22437/jiituj.v10i1.52626

Suryanti, S., Solikhah, B. M., Suliana, R., Pramesti, C., & Sari, A. S. L. (2024). Students' ability to solve story problems on systems of linear equations using John Dewey's approach. *Journal of Education and Learning Mathematics Research (JELMaR), 5*(1), 55–65. https://doi.org/10.37303/jelmar.v5i1.145

Syahputra, E. (2024). Pembelajaran abad 21 dan penerapannya di Indonesia. *Journal of Information System and Education Development, 2*(4), 10–13. https://doi.org/10.62386/jised.v2i4.104

Wahab, A. A., Kusuma, Y. S., Juandi, D., Turmudi, T., Buhaerah, B., & Syaiful, S. (2024). Understanding students' struggles in solving mathematical problems: A systematic literature review using Polya's framework. *Jurnal Pendidikan Progresif, 14*(3), 1728–1753. https://doi.org/10.23960/jpp.v14.i3.2024118

Yamin, Y., Napitupulu, E. E., & Harahap, F. (2022). Development of mathematical LKPD based on scientific approach to improve students' mathematical problem solving at SD Negeri 1 Rimo. *Sensei International Journal of Education and Linguistic, 2*(1), 165–187. https://scispace.com/pdf/development-of-mathematical-lkpd-based-on-scientific-4hqie75gsa.pdf

Yenilmez, K., Özcan, H., Batu, A., & Mart, F. (2022). An analysis of LGS (transition to high school test) mathematics questions in terms of mathematical thinking components. *Osmangazi Journal of Educational Research, 9*(2), 1–21. https://dergipark.org.tr/en/pub/ojer/issue/74184/1136284

Zhao, F. F., & Schuchardt, A. (2021). Development of the sci-math sensemaking framework: Categorizing sensemaking of mathematical equations in science. *International Journal of STEM Education, 8*(10), 1–18. https://doi.org/10.1186/s40594-020-00264-x